Exercise 20

For the following exercises, determine whether there is a minimum or maximum value to each quadratic function. Find the value and the axis of symmetry.

$$f(x) = -\frac{1}{3}x^2 - 2x + 3$$

Solution

Begin by factoring the coefficient of x^2.

$$f(x) = -\frac{1}{3}(x^2 + 6x - 9)$$

In order to write this quadratic function in vertex form, it's necessary to complete the square, which makes use of the following algebraic identity.

$$(x + B)^2 = x^2 + 2xB + B^2$$

Notice that $2B = 6$, which means $B = 3$ and $B^2 = 9$. Add and subtract 9 on the right side within the parentheses and use the identity so that x appears in only one place.

$$\begin{aligned} f(x) &= -\frac{1}{3}[(x^2 + 6x + 9) - 9 - 9] \\ &= -\frac{1}{3}[(x + 3)^2 - 18] \\ &= -\frac{1}{3}(x + 3)^2 + 6 \end{aligned}$$

Therefore, the vertex of the parabola is $(-3, 6)$. The axis of symmetry is $x = -3$, and the maximum (because the coefficient of x^2 is negative) is $y = 6$.